Appendix 3
of Regulation No2 of the Government
of the Republic of 6 January 2011
National Curriculum for Upper Secondary Schools

Last amendment 29 August 2014

Subject field: Mathematics

1. General Principles

1.1. Mathematics Competence

The objective of mathematics education in upper secondary school is development of mathematics competence, which means knowledge of systematic nature of mathematical concepts and relations; ability to use mathematics in the field of mathematics as well as in other subjects and areas of life by modelling various tasks with the help of specific mathematical language, symbols and methods; skill to present problems, identify and implement suitable solution strategies, analyse solutions, verify the accuracy of results; skill of logical reasoning, justification and proof, understanding and use of different solution methods; interest in mathematics and use of links between mathematics and information and communication technology (ICT) tools.

Teaching mathematics is aimed at the upper secondary school graduate having developed the capability to:

1) value mathematics and assess and consider their mathematics competence when planning their career;
2) have acquired a systematic and closely related overview of the terms, relations and procedures of the different areas of mathematics;
3) understand and analyse mathematical texts and present their mathematical trains of thought both orally and in writing;
4) discuss in a creative and logical manner and find and implement suitable strategies to solve open-end problem;
5) formulate mathematical hypotheses and justify and prove them;
6) understand the quantitative, logical, functional, statistical and spatial relations present in the surrounding world;
7) use mathematical methods in other subjects and fields of life, can present a problem in mathematical language and interpret and critically assess mathematical models;
8) interpret different ways of presenting mathematical information (graphs, tables, formulas, diagrams and texts), are able to choose appropriate presentation manners and switch from one manner to another; and
9) use numerous sources of information (models, reference books, ICT tools etc.) in their mathematical activities and critically assess information within them;
10) understand social, cultural and personal significance of mathematics.

1.2. Subjects of the Subject Field and number of courses

The subject domain includes narrow mathematics and extensive mathematics.

The compulsory 8 courses of narrow mathematics are: 'Numerical quantities. Expressions. Equations and inequalities'; 'Trigonometry'; 'Vector on Plane. Equation of a line'; 'Probability and statistics'; 'Functions'; 'Sequences. Function derivative'; 'Integral.Plane figures', 'Stereometry'.

The compulsory 14 courses of extensive mathematics are: 'Expressions. Numerical quantities'; 'Equations and equation systems'; 'Inequalities. Trigonometry Part 1'; 'Trigonometry Part 2'; 'Vector on plane. Equation of a line'; 'Probability and statistics'; 'Functions. Numeric sequences'; ''Exponential and logarithm function**'**; Trigonometric functions'; 'Limit and derivative of function'; 'Applications of derivative'; 'Integral. Planimetry'; 'Line and plane in space'; 'Stereometry'; 'Applications of mathematics and study of actual processes.'

The subject field includes 8 optional courses: 'Logic'; 'Elements of Economical Mathematics'; 'Elements of Number Theory Part 1'; 'Elements of Number Theory Part 2'; 'Elements of Discreet Mathematics Part 1'; 'Elements of Discreet Mathematics Part 2'; 'Planimetry Part 1: Geometry of Triangles and Circles'; and 'Planimetry Part 2: Geometry of Polygons and Circles.'

Elective courses of the subject field can also include courses described under the field of natural science: 'Natural science, technology and society', 'Mechatronics and robotics', '3D modelling', 'Technical drawing', 'Use of computers for inquiry', 'Basics of programming and development of software applications'.

1.3. Description of the Subject Field

Extensive and narrow mathematics differ by both content and approach. Extensive mathematics deals with the terms and methods necessary to understand the nature of mathematics as a science. The purpose of teaching narrow mathematics is observing the application of mathematics in order to describe the world around human beings in a scientific manner and ensure success in life. Students with deeper interest in mathematics can use the study time for elective subjects, nationwide in-depth study forms and individual study. The elective courses specified in the subject syllabus can be added to both narrow and extensive mathematics.

The required environment for this purpose is created by means of dealing with mathematical terms, symbols, properties and relations, laws and procedures and by presenting trains of thought based on intuition and logical discussions. Both narrow and extensive mathematics give pupils the tools and skills to apply appropriate mathematical methods in other subjects.

Students with deeper interest in mathematics can use the study time for elective subjects, nationwide in-depth study forms and individual study.

The obligatory courses included in the syllabus can be added to both narrow and extensive mathematics.

The pupils who study narrow mathematics are allowed to switch to extensive mathematics and the pupils who study extensive mathematics are allowed to switch to narrow mathematics. The conditions of transition are provided in the school curriculum

1.4. Options for forming general competencies

The study of mathematics develops all general competences described in the national curriculum for upper secondary school students. The teacher plays a crucial role in shaping the four interconnected components of competences – knowledge, skills, values and behaviour – with the teacher's values and self-assertion skills creating a suitable learning environment and influencing upper secondary school students' values and behaviour.

Cultural and value competence. By studying mathematics, pupils become familiar with the achievements of mathematicians from different countries and eras, which enables them to recognise connections between different cultures. Pupils are guided to recognise the elegance of logical trains of thought and notice the harmony of geometrical figures in architecture and nature. Persistence, objectivity, preciseness and diligence are developed.

Social and citizenship competence. Responsibility towards society and fellow citizens is raised by solving word problems with relevant content. Various paired and group work assignments help to develop cooperation and mutual assistance skills and enable teachers to use different forms of collective work in mathematics lessons as well. Education increases tolerance towards students with different mathematical abilities.

Self-awareness competence. By means of independent problem solving with varying levels of difficulty, pupils can assess and develop their mathematical abilities. For this purpose, open problem-solving problems are most suitable.

Learning to learn competence. By solving problems, pupils develop analytical, rational method finding and critical outcome assessment skills. Solving of text problems helps to develop functional reading skills: ability to differentiate important and unimportant elements and see connections between objects. They also expand their generalisation and analogy-using skills as well as the skill of using the acquired knowledge in new situations. Pupils are guided towards the understanding that ideas of solution of problems can only be found by using independent thought.

Communication competence. Mathematics develops pupils' ability to express their ideas clearly, briefly and precisely. Above all, this takes place by means of presenting correct definitions, formulating hypotheses and assertions or theorems and also when formulating solutions of problems. By solving word problems, pupils develop their functional reading skills: they learn how to distinguish between the relevant and the irrelevant and see the relations between objects. Mathematics has an important role in developing preparedness to understand, associate and communicate information presented in different ways. Mathematics also expands pupils' ability to formulate information presented in everyday language and vice versa, to present the content of mathematical symbols and formulas in everyday language.

Entrepreneurial competence. New mathematical knowledge is often acquired by means of analysing the characteristics of the objects under observation: pupils study the common properties of the objects, on the basis of which they formulate a hypothesis and come up with ideas to justify the validity of the hypothesis. Education develops the skill of seeing and formulating problems, generating and analysing ideas. Changes caused by different parameters of an object can be examined by solving problems involving probability theory and functions. Identification of different solution paths for a problem improves the flexibility of thought. Finding different solutions to a problem develops pupils' flexible thinking. Entrepreneurial competence is also developed by means of solving various problems involving actual data and through long-term projects.

Natural sciences and technology competence. Studying mathematics is inevitable to use technological aids in solving problems. Mathematics as a language of science allows to understand the significance of science and technology development.

1.5. Options for integrating subjects with other subject fields

The prerequisite for integrating mathematics within a subject includes the order of courses offered in the syllabus. Integrating mathematics with other subject fields and information from outside subjects takes place in implementing the school curriculum.

1.6. Options for implementing cross curricular topics

The cross-curricular topics introduced in the general part of the curriculum are primarily realised in upper secondary school mathematics lessons by systematically organising study activities and the topical content of various problems.

Lifelong learning and career planning.

The study of mathematics through various learning activities helps to shape students' preparation to understand and appreciate lifelong learning as a lifestyle and to interpret career planning as a continuous decision making process. Educational activities offer opportunities for direct contacts with the world of employment, e.g., through visits to undertakings, presentations of occupations, professions and further education opportunities related to the subject field. Education develops independent study skills and responsibility, as well as skills to find and analyse information on further education opportunities, based on one's development needs, and to prepare a career plan. Various learning activities, including independent projects, enable students to link their interests and abilities with subject-specific knowledge and skills and to understand that hobbies can help to balance personal life and career. Realistic assessment of one's abilities is a crucial precondition of further career planning. Mathematics lessons develop the ability of abstract and logical reasoning, which is necessary for weighing different impact factors in career choice. Students develop their studying and communication skills, as well as cooperation, decision making and information handling skills, which are also required in future employment.

Environment and sustainable development. When analysing the environmental resources data, pupils are encouraged to develop a sustainable attitude towards their surroundings and taught to value the physical and social environment. When handling the topic, descriptive mathematics related to percentage calculation, change and relation and elements of statistics are relevant.

Cultural identity. An introduction to the history of mathematics and relations between the development of society and mathematical science are relevant. By means of percentage calculation and statistics, pupils are able to describe the processes occurring in society in relation to the topic of the multicultural world. Geometry plays an important part in the cultural sphere.

Civil initiative and enterprise. Finding different solution paths for problems is associated with enterprise. Initiative and cooperation skills can be developed through research, group work and projects.

Technology and innovation. By means of integrating mathematics courses with technology and natural sciences, pupils can become familiar with the methods of describing and modelling technological processes. Pupils using the ICT solve actual problems and make studying more efficient. Studying

mathematics make opportunities to discover, notice regularities and contribute the development of creative people.

Information environment. Statistics and percentage calculations contribute to an understanding of media manipulations and development of critical information analysis skills.

Health and safety. Problems including health and safety data can teach how to assess risk factors based on objective data.

Values and morality. The study of mathematics develops decency, diligence, systematic approach, persistence, steadiness and integrity. Mathematics plays an important role in developing a tolerant attitude to fellow students with different abilities.

1.7. Planning and Organizing Study Activities

In planning and organising curricular activities:

1) the starting point is basic values, general competences, subject competencies, educational goals and the expected learning outcomess of the curriculum, while also supporting integration with other subjects, generic competencies and cross-curricular topics;
2) the aim is to achieve a moderate learning load (including homework) which is divided evenly during the whole school year and leaves the students enough time for rest and recreational activities;
3) the students are given the opportunity to take part in individual and group learning (individual, paired and group work, educational excursions, practical work, work in computer-based learning environments and with materials from the Internet and other sources of information) to support their development as active and independent learners;
4) differentiated study assignments are used, the essence and level of difficulty of which should support an individualised approach and increase students' motivation to study;
5) learning environments as well as learning materials and equipment based on ICT are used;
6) the educational environment is broadened: computer classes, the school yard, natural environments, museums, exhibitions, enterprises, etc.;
7) the learning process is supported by a wide range of active learning approaches: role play, discussions, debates, project learning, compiling learning folders and research papers, practical work and students` investigations etc.

In planning the learning activities, teachers have a professional right, in cooperation with students, to make choices regarding the presentation of contents with a view of ensuring achievement of required learning outcomes and development of general and field-specific competences, based on students' existing knowledge and skills.

1.8. Assessment

Assessment of learning results is based on the stipulations in the general part of the Upper Secondary School National Curriculum and school curriculum. The assessment forms include formative and summarising assessment.

Formative assessment provides information on general problem solving skills, mathematical reasoning and students' attitude towards mathematics. During lessons or other learning activities, students are given feedback on their knowledge and skills in the subject and subject field, as well as attitudes and values. In cooperation with fellow students and the teacher, a student receives encouraging and constructive feedback on his or her strengths and weaknesses, based on established goals and learning outcomes. Assessment covers knowledge and respective application skills as well as attainment of general competences, incl. studying skills, based on oral answers, written assignments and practical activities. Selection of assessment methods is made in consideration of students' age-specific differences, individual abilities and preparedness to handle specific activities.

The teacher's verbal assessment, numerical grade and the student's self-assessment are all equally important in assessment. Teacher's guidance helps the student to assess his or her performance and results and manage personal development in the subject. Not only the result but also the process is assessed in case of practical assignments and problems. When assessing written assignments, teachers also correct spelling mistakes, but do not take them into account in assessment.

The design and, consequently, assessment of studies is based on hierarchical structure of cognitive processes:

1) Knowledge of facts, procedures and terms: recall, recognition, finding information, calculation, measuring and classifying/ordering.
2) Application of knowledge: choice of methods, presentation of mathematical information in different ways, modelling and solving routine problems.
3) Discussion: reasoning, analysis, synthesis, generalisation, assessment of results and solving actual and non-routine problems.

1.6. Physical Study Environment

1. The school organises education in a classroom which has the necessary tools for drawing on the board.
2. If necessary, the school enables in the mathematics classroom the use of laptop or personal computers with an Internet connection at a ratio of at least one computer to five pupils.
3. The school organises a set of plane and solid figures for classroom use.
4. The school organises a set of pocket calculators for classroom use.

1.9. Physical learning environment

The school provides:

1) a classroom with necessary tools for drawing on the board;
2) if necessary, the classroom should include ICT tools connected to the Internet and with presentation equipment;
3) a set of plane and solid figures for classroom use;
4) a set of pocket calculators for classroom use.

2. Syllabuses

2.1. Narrow Mathematics

2.1.1. Educational and Educational Objectives

Mathematics lessons at the upper secondary school level are designed that students can:

1) understand information presented in mathematical language;
2) use and interpret the various manners of presenting mathematical information;
3) use mathematics to solve problems in different fields of life;
4) value mathematics and find practising mathematics enjoyable;
5) develop their intuition and discuss in a logical and creative manner;
6) use numerous sources of information in their mathematical activities; and
7) use ICT tools when studying mathematics.

2.1.2. Description of the Subject

The purpose of narrow mathematics is to teach pupils to understand information presented in mathematical language and to use mathematics in everyday life situations, thus ensuring the social coping skills of pupils. In narrow mathematics, the teaching process is descriptive and illustrative and the justification of mathematical assertions is based on intuition and analogy. Applied problems form an essential part of the syllabus.

2.1.3. Learning Outcomes in Upper Secondary School

Graduates of upper secondary school:

1) compile and use appropriate mathematical models in order to solve problems in different fields of life;
2) understand and distinguish between functional and statistic processes;
3) simplify expressions and solve equations and inequalities;
4) use trigonometry when solving problems connected with geometrical figures;
5) present basic lines on a plane by means of formulas and sketch the given line on the basis of the formula;
6) use the numerical data describing the probability and distributions of random events by examining the effects of different fields of life;
7) know the properties of functions learnt and use them;
8) find line elements, areas and volumes of geometric figures;
9) express themselves precisely and briefly by using mathematical language and solving problems discuss in a creative and logical manner;
10) use ICT tools when studying mathematics and looking for and processing data;
11) assess their mathematical knowledge and skills and consider them when planning further activities; and
12) know the occupations and professions associated with the subject field, understand the connections between subject field knowledge and labour market opportunities, and analyse their subject-specific knowledge and skills when planning their education.

2.1.4. Learning Outcomes and Learning Content of the Courses

Course 1: 'Numerical Quantities. Expressions. Equations and Inequalities.'

Learning Outcomes

At the end of the course, student can:

1) distinguish between rational, irrational and real numbers;
2) distinguish between equivalence, identity, equation and inequality;
3) explain the identity transformations used to solve equations and inequalities;
4) solve linear, quadratic and simplest fraction equations with one unknown and equations transformable to such equations;
5) perform operations with powers and roots by transforming the latter to powers with rational number exponents;
6) transform simple rational and irrational expressions;
7) solve linear and root inequalities and linear inequality systems with one unknown; and
8) solve simple word problems, including ones based on real life by means of equations and equation systems.

Learning Content

Set of natural numbers N, set of integers Z and set of rational numbers Q. Set of irrational numbers I. Set of real numbers R. Domains of real numbers on a number line. Absolute value of numbers. Simplification of rational expressions. *n*-root of numbers. Generalisation of the term 'power': power with integer and rational exponent. Fractional equation. Presentation of the root of a number as a power with rational number exponent. Operations with powers and examples of operations with roots with equal degree of root. The term 'equation' and its properties. Linear and root inequalities. Solving simple word problems, including ones based on real life by means of equations.

Course 2: 'Trigonometry.'

Learning Outcomes

At the end of the course, student can:

1) define the sine, cosine and tangent of any angle;
2) read graphs of trigonometric functions;
3) transform an angle given in degrees to radians and vice versa;
4) transform simple trigonometric expressions;
5) use the formulas of triangle area and the sine and cosine theorem;
6) solve triangles, calculate areas of a triangle, parallelogram and polygon, calculate an arc of a circumference as a part of the length of the circumference and a sector of a circle as a part of the area of the circle; and
7) solve simple planimetry problems with applied content.

Learning Content

Generalisation of the term 'angle', radian measure. Trigonometric functions ( ) of any angle, and their values if the angles are of 0^0, 30^0, 45^0, 60^0, 90^0, 180^0, 270^0 or 360^0. Trigonometric functions of negative angles. Graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$ functions. Fundamental relations of trigonometry: $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$, $\sin^2\alpha + \cos^2\alpha = 1$, $\cos\alpha = \sin(90^0 - \alpha)$, $\sin\alpha = \cos(90^0 - \alpha)$, $\tan\alpha = \frac{1}{\tan(90^0 - \alpha)}$, $\sin(-\alpha) = -\sin\alpha$, $\cos(-\alpha) = \cos\alpha$, $\tan(-\alpha) = -\tan\alpha$, $\sin(\alpha + k \cdot 360^0) = \sin\alpha$, $\cos(\alpha + k \cdot 360^0) = \cos\alpha$, $\tan(\alpha + k \cdot 360^0) = \tan\alpha$. Sine and cosine

theorem. Formulas for finding an area of a triangle and their application when calculating the area of a polygon. Solving a triangle. Calculating an arc of a circumference as a part of the length of the circumference, and a sector of a circle as a part of the area of the circle. Problems with applied content.

Course 3: 'Vector on a Plane. Equation of Line.'

Learning Outcomes

At the end of the course, student can:

1) explain the term 'vector' and the coordinates of a vector;
2) know a line, circumference and parabola as well as their equations and know the opposite positions of lines on a plane;
3) add and subtract vectors and multiply vectors by a number both geometrically and on the form of coordinate;
4) find the scalar product of vectors and use properties of perpendicularity and collinearity of vectors;
5) compile an equation for a line if the line is determined by a point and a slope, by a slope and a starting ordinate and by two points;
6) determine the mutual positions of lines on a plane;
7) compile an equation for a circumference by a central point and a radius;
8) draw lines, circumferences and parabolas by their equations;
9) find the intersections of two lines (if one of the lines is a straight line); and
10) use vectors and equations for lines in geometry problems.

Learning Content

Determination of a point position on a plane. Distance between two points. The term 'vector' and its symbol. Equality of vectors. Zero vector, unit vector, opposite vector, related vector and free vector. Representation of power as a vector. Coordinates of vectors. Length of vectors. Multiplication of a vector by a number. Addition and subtraction of vectors (geometrically and on a form of coordinate). Angle between two vectors. Scalar product of two vectors and its applications. Collinearity and perpendicularity of vectors. Equation for a straight line (straight line determined by a slope and a starting ordinate, by two points and by a point and a slope). Mutual positions of two lines on a plane. Angle between two straight lines. Equation of a parabola. Equation of r a circumference. Finding the intersections of lines. Equation system compiled by a linear equation with two unknowns, and a linear and a quadratic equation. Problems with applied content.

Course 4: 'Probability and Statistics.'

Learning Outcomes

At the end of the course, student can:

1) distinguish between random, certain and impossible events;
2) understand the concept of event probability and is able to find the number of favourable and all instances (counting, combinatorics);
3) know the nature of the distribution of random variables the meaning of the numerical characteristics of random variables;
4) know the terms 'sample' and 'general dataset' as well as the meaning of reliability of data classification and statistical decision;
5) calculate the probability of an event and use it when solving simple problems connected with actual data;

6) calculate the numerical characteristics of a random event and draw conclusions regarding the problem under observation;
7) find the mean confidence region of a general dataset on the basis of a sample; and
8) collect data and analyse it by statistical means of ICT.

Learning Content

Event. Types of events. Relative frequency and statistical probability. Classical probability. Geometric probability. Product of events. Probability of product of independent events. Sum of events. Probability of sum of exclusive events. Factorial. Permutations. Combinations. Binomial coefficient. Discreet random variable, its distribution law, distribution polygon and numerical characteristics (mean, mode, median and standard deviation). General dataset and sample. Collection of data and data classification. Analysis of dataset by one criterion. Normal distribution (descriptively). Reliability of statistical decisions based on mean of confidence region. Data processing project compiled bu ICT tools (recommended as cooperation project with other subject).

Course 5: 'Functions.'

Learning Outcomes

At the end of the course, student can:

1) explain the term and general symbol of a function, the terms connected with the investigation of functions, and know the terms of inverse function and odd and even function;
2) draw graphs of the functions determined by the syllabus (both by hand and on computer);
3) describe the main properties of a function by means of the graph of function;
4) know the term and properties of the logarithm of a number, find the logarithm and potentiate simpler expressions;
5) solve simpler exponent and logarithmic equations by means of direct use of the definitions of the power and logarithm;
6) understand the nature of increase and decrease by compound interest and use it to solve simpler problems involving actual data;
7) interpret quantities expressed in percentages in other subjects and in real life; and
8) solve main trigonometric equations in a given interval on the basis of a graph.

Learning Content

Functions $y = ax + b$, $y = ax^2 + bx + c$ and $y = \frac{a}{x}$ (review). Term and general symbol of a function Methods presentation of functions. Domain and range of a functions. Even and odd functions. Zeros of a function, Domain of positivity and negativity of a function. Increase and decrease of a function. Extreme of functions. Functions $y = ax^n$ $(n = 1, 2, -1\ ja - 2)$. The term 'logarithm of a number'. Logarithm of a product, quotient and power. Finding the logarithm and potentiating (to an extent that enables pupils to solve simpler exponent and logarithmic equations). Inverse function. Functions $y = a^x$ and $y = \log_a x$. Increase and decrease by compound interest. Examples of models containing e^{ax}. Simpler exponent and logarithmic equations. The terms *arcsin m*, *arccos m* and *arctan m*. Examples of finding solutions to main trigonometric equations.

Course 6: 'Sequences. Function derivative'

Learning Outcomes

At the end of the course, student can:

1) understand the terms 'numeric', 'arithmetic' and 'geometric sequence';
2) use the formulas of the general member of arithmetic and geometric sequence and the sum of the first nth members for simple problems based on real life;
3) explain the terms 'derivative of a function' and 'tangent of the function graph' and the geometric meaning of derivative of a function;
4) find the derivative of functions;
5) compile the equation for the tangent of the function graph at a given point of tangency;
6) explain the relation of increase and decrease of function with the derivative of a function, the term extremum of a function and the finding the extremum;
7) find, as specified in the subject syllabus, the zeros, domains of positivity and negativity, domains of increase and decrease, maximum and minimum points of simple functions and draw a graph of the function on the basis of this information; and
8) solve simple extremum problems.

Learning Content

The term 'number sequence' and general member of a sequence. Arithmetic sequence, formula for its general member and sum. Geometric sequence, formula for its general member and sum.
Geometrical meaning of the derivative of a function. Slope of the tangent of a line and equation of tangent.

Derivatives of the functions $y = x^n (n \in Z)$, $y = e^x$ and y = ln x. Derivates of the sum, difference, product and quotient of functions. Second derivative of a function. Study of the increase and decrease of a function and using the derivative to find the extreme points. Simplest extremum problems.

Course 7: 'Planimetry. Integral.'

Learning Outcomes

At the end of the course, student can:

1) recognise the geometric figures determined by the syllabus and explain the main properties of the figures;
2) use the terms of geometry and trigonometry and their main relations when solving problems connected with actual data;
3) recognise the term 'antiderivative' and find indefinite integrals (from polynomials);
4) recognise the term area under graph' and use the Newton-Leibniz Formula to calculate a definite integral; and
5) calculate the area of plane figures on the basis of a given integral.

Learning Content

Triangles, quadrangles, regular polygons, circumference and circle. Properties of these objects, relations between elements, perimeters and areas in problems with applied content.
Antiderivative and indefinite integral. Definite integral. Newton-Leibniz Formula. Area under graph and its area. Integration of simple functions. Calculation of the area of a plane figures on the basis of a definite integral. Problems with applied content.

Course 8: 'Stereometry.'

Learning Outcomes

At the end of the course, pupils can:

1) describe the coordinates of a point in space and the mutual positions of the straight lines and planes in space;
2) explain the terms of angles between two straight lines, a line and a plane and two planes;
3) recognise the properties of solids and solids of revolution determined by the syllabus and the process of calculating their area and volume;
4) depict solid figures on a plane and their simple cross sections with a plane (for instance, axial cross section, parallel cross section with a single face);
5) calculate the line elements, areas and volumes of the solids determined by the syllabus;
6) use knowledge in trigonometry and planimetry to solve simple stereometrical problems and
7) use solid figures as models to solve problems connected with actual data.

Learning Content

Cartesian coordinates in t space. Coordinates of a point. Distance between two points. Mutual positions of two lines in t space. Angle between two straight lines. Mutual positions of a straight line and a plane in space. Angle between a straight line and a plane. Properties of perpendicularity between a straight line and a plane. Mutual positions of two planes in space. Angle between two planes. Prism and pyramid. Total area and volume of vertical prisms and regular pyramids. Cylinder, cone and sphere, their total area and volume. Examples of sections of solid figures with a plane. Practical problems on polyhedrons (vertical prism and pyramid) and solids of revolution.

2.2. Extensive Mathematics

2.2.1. Educational and Educational Objectives

Mathematics lessons at the upper secondary school level are designed that students can:

1) understand information presented in mathematical language and present their mathematical trains of thought both orally and in written form;
2) choose, interpret and associate various manners of presenting mathematical information;
3) discuss in a logical and creative manner and develop their intuition;
4) formulate mathematical hypotheses, argue and prove them;
5) model problems in different fields of life in a mathematical manner and assess mathematical models critically;
6) value mathematics and find practising mathematics enjoyable;
7) use different sources of information in their mathematical activities and critically assess the information within them; and
8) use ICT tools for study purposes.

2.2.2. Description of Subject

Extensive mathematics introduces the meaning of mathematics in the development of society and gives examples of its application in everyday life, technology, economy, natural and exact sciences and other fields of society. In order to achieve this objective, numerous problems with applied content are solved by

means of a computer and appropriate ICT software. Argumentation and proving hold an important place in these studies.

2.2.3. Learning Outcomes in Upper Secondary School

Graduates of upper secondary school:

1) understand and use the mathematical methods and procedures learnt;
2) discuss in a logical and creative manner and formalise their mathematical trains of thought;
3) understand and distinguish between functional and statistic processes;
4) compile and use appropriate mathematical models in order to solve problems in different fields of life;
5) use different ICT tools for study purposes;
6) transform irrational and rational expressions and solve equations and inequalities and equation and inequality systems;
7) transform trigonometric expressions and use trigonometry and vectors to solve problems of geometry;
8) compile equations for a line and draw known lines on the basis of their equations;
9) use numerical data describing the probability of random events and distributions by studying the effects of different fields of life;
10) study functions on the basis of derivative; and
11) know the properties of plane and solid figures and find the areas and volumes of geometrical objects (also by means of an integral).

Course 1: 'Expressions and Numerical Quantities.'

Learning Outcomes

At the end of the course, student can:

1) explain the properties of the set of natural numbers N, set of integers Z, set of rational numbers Q, set of irrational numbers I, and set of real numbers R;
2) define the absolute value of numbers;
3) mark the regions of real numbers on a number line;
4) present the root of a number as a power with a rational exponent and vice versa;
5) perform operations with powers and roots with equal degrees of root;
6) transform simpler rational and irrational expressions; and
7) solve problems with applied content (including percentage problems).

Learning Content

Set of natural numbers N, set of integers Z, set of rational numbers Q, set of irrational numbers I and set of real numbers R; their properties. Regions of real numbers on a number line. Absolute value of numbers. Numerical systems (using the example of the binary system). Rational and irrational expressions. *n*-root of numbers. Generalisation of the term 'power': power with integer and rational exponent. Operations with powers and roots.

Course 2: 'Equations and Equation Systems.'

Learning Outcomes

At the end of the course, student can:

1) explain the terms ‘equivalence’, ‘identity’, ‘equation’, ‘s root of an equation’, ‘set of solutions of a system of equations and inequalities’ ;
2) explain the identity transformations used to solve equations and system of equations;
3) solve linear, quadratic, fractional and simplest r irrational equations with one unknown and equations transformable to such equations;
4) solve simple equations with one absolute value;
5) solve systems of equations; and
6) solve word problems by means of equations and systems of equations.

Learning Content

Equivalence, equation and identity. Equivalent equations and identity transformations. Linear, quadratic, fractional and irrational equations (up to two roots) and equations transformable to such equations. Equation with one absolute value. Systems of equations. Two and three dimensional determinant. Word problems.

Course 3: ‘Inequalities. Trigonometry Part 1.’

Learning Outcomes

At the end of the course, student can:

1) explain the properties of inequalities and the terms ‘inequality’ and ‘set of solutions of a system of inequalities ;
2) explain the identity transformations used to solve inequalities and systems of inequalities;
3) solve linear, quadratic and fractional inequalities and simple systems of inequalities;
4) use a pocket calculator to find the values of trigonometric functions of an acute angle and by these values the size of the angle;
5) solve a right-angled triangle;
6) use trigonometric functions of a supplementary angle; and
7) use main trigonometric relations in simplification problems.

Learning Content

The term ‘inequality’ and its properties. Linear inequalities. quadratic inequalities. Interval method. Simpler fractional inequalities. Systems of inequalities.
Sine, cosine and tangent of an acute angle. Trigonometric functions of a supplementary angle.
Main trigonometric relations in a right-angled triangle.

Course 4: ‘Trigonometry Part 2.’

Learning Outcomes

At the end of the course, student can:

1) transform an angle given in degrees to radians and vice versa;
2) calculate the length of an arc of a circumference as a part of the circumference and the area of a sector of a circle as a part of the circle;
3) define the sine, cosine and tangent of any angle and deduce the relations between sine, cosine and tangent;
4) deduce and know the exact values of sine, cosine and tangent of 0˚, 30˚, 45˚, 60˚, 90˚, 180˚, 270˚ and 360˚, use reduction formulas and the formulas for negative and bigger than round angles;

5) use a pocket calculator to find the values of trigonometric functions and use the values to find the size of an angle;
6) know the sum and difference formulas of two angles and deduce and know the formulas of sine, cosine and tangent of a double angle;
7) transform simple trigonometric expressions;
8) prove the sine and cosine law;
9) solve a triangle and calculate its area; and
10) uset trigonometry to solve problems in different fields of life.

Learning Content

Generalisation of the term 'angle'. Degree measure and radians measure of an angle. Trigonometric functions of any angle. The exact values of sine, cosine and tangent of 0º , 30º , 45º, 60º, 90º, 180º, 270º and 360º. Relations between the trigonometric functions of one and the same angle. Reduction formulas. Trigonometric functions of negative and bigger than round angles. Trigonometric functions of a sum and a difference of two angles. Trigonometric functions of double angles. Trigonometric expressions. Length of the arc of a circumference and the area of a sector of a circle. Formulas for area of a triangle. Sine and cosine laws. Solving triangles. Problems with applied content.

Course 5: 'Vector on Plane. Equation of Line.'

Learning Outcomes

At the end of the course, student can:

1) explain the terms 'vector'. 'unit, zero and opposite vector'. 'coordinates of a vector' and 'angle between two vectors';
2) add, subtract and multiply vectors by a number both geometrically and on the form of coordinates ;
3) calculate the scalar product of two vectors and use vectors in problems with physical content;
4) use the perpendicularity and collinearity properties of vectors;
5) solve triangles by means of vectors;
6) find the coordinates of the centre of a line segment;
7) deduce and compile an equation for a straight line (when the line is determined by a point and a vector parallel to the line, a point and a slope, a slope and an initial ordinate and two points) and transform it to a general equation, determine the mutual position of two lines on a plane and in the case of intersecting lines find the cross point of the lines and the angle between the lines; and
8) compile equations for a hyperbola, parabola and circumference, draw lines determined by the syllabus on the basis of their equations and find the cross points of two lines.

Learning Content

Distance between two points. The term 'vector' and its symbols. Zero vector, unit vector, opposite vector, bound vector and free vector. Equality of vectors. Coordinates of vectors. Length of vectors. Addition and subtraction of vectors. Multiplication of a vector by a number.

Coordinates of the centre of a line segment. Angle between two vectors. Collinearity of vectors. Scalar product of two vectors, its applications and perpendicularity of vectors. Solving triangles by means of vectors.

Equation of a line. General equation of a line. Mutual positions of two lines on a Plane Angle between two lines. Equation for a circumference. Parabola $y = ax^2 + bx + c$ and hyperbola $y = \frac{a}{x}$. The term 'equation of a line'. Intersection of two lines.

Course 6: 'Probability and Statistics.'

Learning Outcomes

At the end of the course, student can:

1) distinguish between random, certain and impossible events and explain the terms, types and properties of the probability of an event;
2) explain the meaning of permutations, combinations and variations and find their number;
3) explain the meaning of the product of dependent and independent events and the meaning of the sum of exclusive and non-exclusive events;
4) calculate the probability of different events, including ones related to real life;
5) explain the nature of distribution of of random variable and the meaning of the numerical characteristics (mean, mode, median and standard deviation) of random variables, describe binomial and normal distributions and use Bernoulli's Formula to calculate probability;
6) explain the terms 'sample' and 'general dataset' as well as the meaning of reliability of data classification and statistical decision;
7) calculate the numerical characteristics of a random variable and draw conclusions regarding the distribution or problem under observation;
8) find the average region of confidence of the general dataset on the basis of a sample; and
9) collect data and analyse it by ICT through statistical means.

Learning Content

Permutations, combinations and variations. Event. Types of events. Classical probability. Relative frequency and statistical probability. Geometric probability. Types of events: dependent and independent, exclusive and non-exclusive. Addition and subtraction of probabilities. Bernoulli's Formula. Discreet and continuous random variable, binomial distribution, distribution polygon and numerical characteristics (mean, mode, median, dispersion and standard deviation). Problems with applied content.

General dataset and sample. Collection of data and data classification. Analysis of statistical data by one criterion. Correlation table. Linear correlation coefficient. Normal distribution (by example). Reliability of statistical decision on the basis of the mean of a region of confidence. Data processing project compiled on a computer (recommended as a cooperation project with another subject).

Course 7: 'Functions. Number Sequences.'

Learning Outcomes

At the end of the course, student can:

1) explain the term and general symbol of a function and the terms related to the investigation of functions;
2) describe the properties of graphically presented functions and sketch graphs by hand and by means of computer programmes;
3) find the domain of definition, zeros, domains of positivity and negativity of a function presented algebraically and check whether the function is odd or even;

4) describe the relation between the graph of function $y=f(x)$and the graphs of functions $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$ and $y = a f(x)$;
5) explain the terms 'number', 'arithmetic' and 'geometric sequence' and 'vanishing geometrical sequence';
6) deduce the formulas of the sum of the first *n*-member of the arithmetic and geometric sequence and the sum of the vanishing geometric sequence and apply them and the formulas of the general member of the arithmetic and geometric sequence to solve problems;
7) explain the nature of the limit of a sequence and calculate the limit and know the meaning of numbers π and *e*; and
8) solve problems involving actual data on the basis of arithmetic and geometric sequence and the vanishing geometric sequence.

Learning Content

Functions $y = ax + b$, $y = ax^2 + bx + c$ and $y = \frac{a}{x}$ (review). The term and general symbol of a function. Ways for present functions. Domain and range of functions. Even and odd functions. Zeros of functions and domains of positivity and negativity of a function. Domains of increase and decrease of a function. Extreme of functions. Power function. Graphs and properties of the functions $y = x$, $y = x^2$, $y = x^3$, $y = x^{-1}$, $y = \sqrt{x}$, $y = \sqrt[3]{x}$, $y = x^{-2}$, and $y = |x|$. Composite function. Inverse function. Graphs of the functions $y = f(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$, $y = a f(x)$ on a computer.
The term 'number sequence', general member of the sequence and types of sequences. Arithmetic sequence and its properties. The formula for the general member of an arithmetic sequence and the formula for the sum of the first *n*-member. Geometric sequence and its properties. The formula for the general member of a geometric sequence and the formula for the sum of the first *n*-member. Limit of a number sequence. Calculation of the limit. Vanishing geometric sequence and its sum. Number *e* as a limit. Length of a circumference and area of a circle as a limit and number π. Problems with applied content.

Course 8: 'Exponential and logarithm function'

Learning Outcomes
At the end of the course, student can:
1) explain the nature of increase and decrease by composite percentage;
2) solve tasks involving increase and decrease by composite percentage;
3) describe the properties of exponential function, including the function $y = e^x$;
4) explain the term and properties of the logarithm of a numeral and find the logarithm and potentiate elementary expressions, change logarithm base;
5) describe the logarithm function and its properties;
6) find the inverse of exponential and logarithmic functions;
7) draw the graphs of exponential and logarithm functions and read the properties of the functions from the graph;
8) solve elementary exponent and logarithm equations and inequalities;
9) use exponential and logarithm functions to model and examine real life effects.

Learning Content

Increase and decrease by composite percent. Exponential function, its graph and properties. Logarithm of a number. Logarithm of a product, quotient and power. Finding the logarithm and potentiating. Transformation from one basis of a logarithm to another. Logarithm function, its graph and properties. The concept of inverse function, based on the example of exponential and logarithmic function. Exponent and logarithm equations and solving such equations. Problems with applied content on exponential and logarithm functions. Exponent and logarithm inequalities.

Course 9: 'Trigonometric functions. Limit and Derivative of Function.'

Learning Outcomes

At the end of the course, student can:

1) explain the term of periodicity of a function and find the period of sine, cosine and tangent functions;
2) draw graphs of sine, cosine and tangent functions and read the properties of the functions from the graphs;
3) find the general and particular solutions of simple trigonometric equations in a given area and solve simple trigonometric inequalities;
4) explain the terms 'limit of a function' and ' derivative of a function' and the physical and geometric meaning of the derivative;
5) present composite functions through simpler functions;
6) apply the rule of calculating sum, difference, product and quotient of functions, find the first and second derivative of a function.

Learning Content

Periodicity of a function. The graphs of sine, cosine and tangent functions and their properties. The terms *arcsin m, arccos m* and *arctan m*. Simple trigonometric equations.

Limit and continuity of a function. Increment of an argument and increment of a function. Instantaneous velocity. Slope of the tangent of the function graph. The term 'derivative of a function'. Geometric meaning of the derivative of a function. Derivative of the sum and difference of functions. Derivative of the product of two functions. Derivative of the quotient of two functions. Composite function and finding derivatives of composite functions. ~~Derivative of a power function.~~ Derivative of a composite function. Second derivative of a function. Derivatives of trigonometric functions. Derivatives of exponential and logarithm functions. Table of derivatives.

Course 10: 'Applications of Derivatives.'

Learning Outcomes

At the end of the course, student can:

1) compile the equation of the tangent of the graph of a function;
2) explain the relation of increasing and decreasing of a function with the sign of the derivative of a function, the term extremum of a function and finding the extremum points;
3) find the domains of increase and decrease and extreme points of functions, convexity and concavity intervals of graphs of function and the point of inflexion;
4) investigate functions specified in the syllabus thoroughly and sketch a graph based on the properties of the function;
5) find the greatest and smallest value of the function in the given segment; and
6) solve extremum problems with applied content (including those with economic content).

Learning Content
Slope of a tangent. Equation of a tangent of a line. Domains of increasing and decreasing of a function; extremum of a function; necessary and sufficient condition for the existence of extremum. The greatest and smallest value of a function in a given line segment. Convexity and concavity intervals of function graphs and the point of inflexion. Investigation of a function by means of derivative. Sketching a graph based on the properties of the function. Problems with applied content (use of derivative of a function). Extremum problems.

Course 11: 'Integral. Review of Planimetry.'

Learning Outcomes
At the end of the course, student can:

1) explain the term 'antiderivative' and find the indefinite integrals of simple functions by means of the table of main integrals and properties of an integral;
2) explain the term ' area under graph' and apply the Newton-Leibniz Formula to calculate the definite integral;
3) calculate the area of an area under graph by means of a definite integral, the area of a part of surface made up of several parts and a part of surface limited by two curves and a volume of an simple solid of revolution;
4) explain the properties of geometric figures and their elements, image the corresponding figures in a drawing, use a ICT tools to investigate the properties of geometric figures and image the corresponding figures in a drawing;
5) explain the congruence and similarity properties of triangles, the properties of similar polygons and calculation of the perimeter and area of these figures;
6) solve calculation problems in planimetry (also simple proving problems); and
7) use geometric figures as models to investigate the objects in the surrounding area.

Learning Content
The terms 'antiderivative' and 'indefinite integral'. Properties of integrals. Area under graph and its area as a limit. Definite integrals and Newton-Leibniz Formula. Use of integral to calculate the area of a plane figures, the volume of a solid of revolution, a polyhedron and work.

Triangle and its interior and exterior angles, bisector of an interior angle of a triangle and its properties. Interior and exterior circumference of a triangle. Median of a triangle and properties of median. Midline of a triangle and its properties. Metric relations in a right-angled triangle. Polygon and its types. Sum of interior angles of a curved polygon. Similarity of polygons. Relation of perimeters and areas of similar polygons. Interior and exterior circumference of a polygon. Parallelogram, its special types and properties. Trapezoid and its types. Midline of a trapezoid and its properties. Central angle and angle at the circumference. Thales' Theorem. Secant and tangent of a circumference. Chord and tangent polyhedrons. Area of a triangle. Geometrical problems with applied content.

Course 12: 'Line and Plane in Space.'

Learning Outcomes
At the end of the course, student can:

1) describe a point in space with the coordinates;

2) explain the term ‘spatial vector’, linear operations with vectors, collinearity and complanarity of vectors and the scalar product of vectors;
3) describe the mutual positions of a line and a plane;
4) calculate the distance between two points, the length of a vector and an angle between two vectors;
5) determine the position of two given straight lines, a straight line and a plane and two planes and calculate the angle between them in stereometry exercises; and
6) use vectors to solve problems with geometric and physical content.

Learning Content
Position propositions in spatial geometry: an angle between two lines, a line and a plane and two planes, perpendicularity and parallelism of the lines and planes, the theorem of three perpendiculars and the area of the projection of a polygon.
Cartesian coordinates of a point in space. Coordinates of a point in space and the position vector of a point. Coordinates of a vector in space and length of a vector. Linear operations with vectors. Collinearity and coplanarity of vectors, expressing a vector by any three non-complanar vectors. Scalar product of two vectors. Angle between two vectors.
Equations for a line in space and the equation for a plane. Investigation of the mutual positions of the lines and planes given by equations in space, cross point of a line ´and a plane and finding an angle between the lines given by equations. Problems with applied content.

Course 13: ‘Stereometry.’

Learning Outcomes
At the end of the course, student can:
1) know the types of polyhedrons and solids of revolution and the formulas for calculating their areas;
2) image a prism, a pyramid, a cylinder, a cone and a sphere on a drawing and their simple cross sections with a plane;
3) calculate the area and volume of solids and the area of the cross section of the solids and a plane; and
4) use polyhedrons and solids of revolution as models to investigate the objects in the surrounding area.

Learning Content
Prism and pyramid, their area and volume and regular polyhedrons. Solids of revolution, a cylinder, a cone and a sphere, their area and volume and a segment, layer, zone and sector of a sphere. Deduction of a formula for calculating the volume of a cylinder, a cone or a sphere. Problems for polyhedrons and solids of revolution. Cross sections of polyhedrons and solids of revolution with a plane. Problems with applied content.

Course 14: ‘Applications of Mathematics and Exploration of Actual Processes.’

Learning Outcomes
At the end of the course, student can:
1) explain the general nature of mathematical modelling and its procedures;
2) know the methods and functions necessary to compile simple models;
3) use more important models and methods of natural and economic sciences;

4) solve word problems by means of equations;
5) know the regularities and relations between various fields of life in the surrounding world;
6) compile mathematical models of easy-to-model real effects and use them to study reality; and
7) use ICT tools for solving problems.

Learning Content
Meaning of mathematical models, stages of modelling an effect and assessment of the quality and practicability of models. Solving word problems (including percentage problems) by means of equations as in compiling and solving mathematical models of the problems.
Models in natural and economic sciences, technology and other fields (such as the relations between physical variables, models of organic growth in biology, demand and supply functions and marginal functions in economic science and calculations of material consumption in technology etc.) which use linear, quadratic and exponent functions. The approach of the course is based on the use of calculation tools.

3. Syllabuses of optional courses

3.1. Optional Course: 'Logic.'

Learning and Educational Objectives

The objective of the optional course is that students:
1) have achieved an overview of the historical development of logic and of its fields of use;
2) define terms correctly and are able to correct inaccurate definitions;
3) realise the necessity of proving and are able to use appropriate mathematical tools;
4) determine truth values (knowing the truth values of component statements) on the basis of the truth values of component statements; and
5) explain the formation of paradoxes.

Short Description of Course
The course includes terms known to pupils since basic school (definition, theorem, assumption and assertion). However, some new terms are added (types of theorems, quantifiers, propositions and paradoxes). Special attention is given to presenting a mathematical text by means of quantifiers and to determining the truth value of simple proposition. Pupils analyse well-known paradoxes and examine the formation process of paradoxes.

Learning Outcomes
At the end of the course, student can:
1) determine the content and extent of the term and classify terms;
2) define the terms and find inaccuracies and mistakes in given definitions;
3) separate the assumption and assertion of a theorem and compile converse, inverse and contrapositive theorems on the basis of the given theorem and prove the theorem;
4) use quantifiers to write a mathematical text;
5) perform operations with propositions and determine the truth value of a proposition; and
6) explain the formation of paradoxes.

Learning Content

What does logic teach? Historical background. The term. Definition and classification of a term. Decision. Propositions in logic. Truth value of a proposition. Operations with propositions. Negation. Disjunction and conjunction. Implication. Equivalence. Compound proposition and finding their truth value by means of the truth table. Laws of logic. Law of double negation. Law of contradiction. Law of excluded middle. Consequence relations. Proving. Axiom. Theorem. Converse of a theorem. Inverse of a theorem. Contrapositive of a theorem. Proof by contradiction.. Necessary and sufficient conditions. Paradoxes.

3.2. Optional Course: 'Elements of Economical Mathematics.'

Learning and Educational Objectives

The objective of the optional course is that students:

1) become familiar with the fundamental mathematical models describing the operation of the economic world around them and the methods of their implementation; and
2) are able to use mathematics to make reasonable decisions in their economic activities.

Description of Course

The course consists of three basic fields:

1) use of percentage calculation to solve economic problems (indexes, taxation, price formation and calculations involving different currencies);
2) modelling of economic processes by means of functions (demand, offer, expenses, income, net income, advertising income and ordering of goods); and
3) basics of financial mathematics (interest, fines and loans).

Learning Outcomes

At the end of the course, student can:

1) explain the nature of price indexes and their calculation as an application of percentage calculation;
2) use percentage calculation to calculate and interpret price indexes, including consumer price indexes;
3) explain the meaning of primary tax types (income tax, social tax, sale tax and excise tax etc.) and their calculation processes as the applications of percentage calculation;
4) use percentage calculation to explain and find income expenses and the development of the price of goods (simpler cases);
5) explain the basic terms related to money and currency (exchange rate, conversion, inflation and real income) and are able to find and calculate them in simpler cases;
6) explain the use of functions in order to model demand, offer, market balance, expenses, income, net income and advertising income and are able to compile and implement these models in simpler cases (primarily linear models); and
7) explain the terms 'simple' and 'compound interest' and are able to use them in order to manage situations involving deposits and loans (paying bills, fines, deposit profitability, expenses and write-off of loans using the example of study and housing loans).

Learning Content

Basic problems for percentage calculation. Indexes. Consumer price index. Primary taxes using the example of income, social, value added and excise taxes. Income expenses. Development of the price of goods. Exchange rate and conversion. Calculation of inflation by means of the consumer price index. Real

income. Demand and offer functions. Market balance. Expense, revenue and net revenue functions. Advertising revenue function. Simple and compound interest. Bills and fines. Deposit profitability. Expenses of loans using the example of study and housing loans.

3.3. Optional Course: 'Elements of Number Theory Part 1.'

Learning and Educational Objectives

The objective of the optional course is that students:

1) achieve a better concept of integer presentation in the decimal system as well as the relations between numbers, main results and proving methods relevant in contemporary computer science and other fields of life;
2) understand and are able to use basic proving methods to prove main results and solve problems; and
3) develop creative and flexible mathematical thinking.

Description of Course

The course consists of four basic fields:

1) presentation of an integer in the decimal system;
2) divisibility of integers and remainder arithmetic ;
3) prime and composite numbers and the fundamental theorem of arithmetic; and
4) classes of numbers with special properties.

Learning Outcomes

At the end of the course, student can:

1) use for solving problems the appropriate presentation of an integer in the decimal system as the sum of multiplies of power of ten;
2) define the term 'integer divisibility' and prove the basic properties of divisibility relation;
3) use the basic properties of divisibility to deduce divisibility properties and solve classical (proving)problems;
4) define the term of division with a remainder and prove the main statements of remainder arithmetic;
5) use remainder arithmetic to solve classical (proving)problems;
6) define the terms of a prime and a compound number and the greatest common divisor and the least common multiple of two integers;
7) formulate (and prove if possible) the fundamental theorem of arithmetic and use it to solve classical (proving)problems;
8) explain the algorithms used to find the greatest common divisor and the least common multiple of the integers and use them to solve (proving)problems; and
9) present an overview of the origin and properties of numbers from the so-called 'interesting' type of numbers (such as triangular numbers, amicable numbers etc.).

Learning Content

Presentation of integers in decimal system: presentation of an integer as the sum of multiplies of power of ten. The last numbers of an integer and the decimal presentation of its powers.

Divisibility of integers and remainder arithmetic. Divisibility. Properties of the divisibility relation. Division with a remainder. Divisibility properties.
Multiples and divisors of numbers. Prime and composite numbers. Greatest common divisor and least common multiple. Fundamental theorem of arithmetic. Interesting numbers. Polygonal, ideal and amicable numbers etc.

3.4. Optional Course: 'Elements of Number Theory Part 2.'

Learning and Educational Objectives

The objective of the optional course is that students:

1) achieve a better understanding of the different possibilities and principles of the composition of the number systems currently in use and the basic terms and results of the number theory used in computer science;
2) understand and are able to use various proving methods to prove results and solve proving problems; and
3) develop creative and flexible mathematical thinking.

Description of Course

The course consists of five basic fields:

1) principle of mathematical induction;
2) congruencies;
3) number systems;
4) canonical presentation of rational numbers; and
5) Euclidean Algorithm.

Learning Outcomes

At the end of the course, student can:

1) explain the nature of mathematical induction and its implementation possibilities and use the principle of mathematical induction to solve (proving) problems of varying difficulty;
2) define the remainder equality or the congruence of integers by modulusand prove the main properties of congruences;
3) use congruences to solve (proving)problems;
4) explain the different principles of the construction of number systems and give historical examples about different systems;
5) transform the numbers of the decimal system into the numbers of another number system and vice versa and perform operations with the numbers from a system with a base different from ten;
6) present a natural number in a canonical form and find the number of its positive divisors and the sum of its divisors;
7) know the presentations of a rational number as an irreducible fraction and in a canonical form and use them to solve problems;
8) use the Euclidean Algorithm to find the greatest common multiple and least rational number as a continuous fraction; and
9) solve linear Diophantine equations with two unknowns.

Learning Content

Principle of mathematical induction: the principle and its use in solving problems.
Congruences: congruence of integers by means of a modulus Main properties of congruences. Use of congruences in (proving) problems in number theory.
Number systems: positional and non-positional number systems. Examples of number systems with different bases and their construction principles.
Canonical presentation: canonical presentation of a positive integer and its applications. Presentation of a rational number as an irreducible fraction and in a canonical form.
Euclidean Algorithm: finding the Greatest common divisor. Solving of linear Diophantine equations with two unknowns. Presentation of a rational number as a continuous fraction.

3.5. Optional Course: 'Elements of Discrete Mathematics, Part 1.'
Learning and Educational Objectives

The objective of the optional course is that students:

1) achieve an overview of the problems of the quickly developing and important field of contemporary mathematics, discreet mathematics, and their primary solving methods (including Dirichlet's Principle and invariants);
2) are able to use argumentation and proving methods peculiar to discreet mathematics to solve simple (proving) problems and formulate solutions correctly r; and
3) develop creative and flexible mathematical thinking.

Description of Course

The course consists of three basic fields:

1) logic problems;
2) Dirichlet's Principle; and
3) method of invariants.

Learning Outcomes

At the end of the course, student can:

1) solve simple, typical logic problems determined by the syllabus by means of appropriate tables, schemes and drawings where necessary;
2) formulate Dirichlet's Principle and prove it by contradiction;
3) use Dirichlet's Principle and its generalisation to solve word and simple (proving) problems connected with numbers;
4) formulate the analogue of Dirichlet's Principle in geometry and use it to solve simple planimetry problems;
5) use Dirichlet's Principle to solve simpler (proving) problems by colouring the parts of a plane;
6) explain the nature of the method of invariants and are able to name invariants related to integers (such as parity, sums of numbers, products and remainders); and
7) solve simple problems connected with games and tables of numbers by means of choosing the appropriate invariant.

Learning Content

Logic problems (finding the correspondence between elements of a set): "who's who" type problems, problems to determine those telling the truth and those lying and problems connected with weighing and pouring.

Dirichlet's Principle: the nature of the principle and its proving by contradiction. Generalisation of the principle. Application of the principle to solve word problems and problems of number theory. Analogue of Dirichlet's Principle in geometry and its use to solve simple geometry and colouring problems.
Invariants: the use of invariants related to parity and other integers in simple problems connected with games and tables of numbers.

3.6. Optional Course: 'Elements of Discrete Mathematics Part 2.'

Learning and Educational Objectives

The objective of the optional course is that students:

1) achieve an overview of the simpler problems of the two fields of the quickly developing and important discrete mathematics, combinatorics and graph theory, and their solving methods;
2) are able to use argumentation and proving methods peculiar to discrete mathematics to solve (proving) problems and formulate solutions correctly ; and
3) develop creative and flexible mathematical thinking.

Description of Course

The course consists of three basic fields:

1) mathematical induction;
2) elements of combinatorics; and
3) introduction to graph theory.

Learning Outcomes

At the end of the course, student can:

1) formulate the classical version of the principle of mathematical induction (by step 1) and explain the importance of a basis and a step of induction;
2) use the principle of mathematical induction to solve (proving) problems of various difficulty;
3) formulate the main rules of combinatorics (addition and multiplication rule) and explain their nature and use the main rule appropriate to the context of the (proving) problems to calculate the options of the object;
4) define unions without repetitions (permutations, variations and combinations) and deduce the formulas to find their number and use them to solve (proving) problems on the basis of the context of the problem;
5) explain the term 'unions with repetition' (permutations, variations and combinations) and use their calculation formulas to solve simpler problems;
6) know the term 'graph' and the basic related terms and methods (edge, vertex, degree of vertex and counting of edges) and solve problems with relevant content;
7) formulate and prove a theorem on the number of odd-numbered vertexes of a graph and use it to solve simple problems;
8) know types of graphs (Euler graph, coherent graph, tree graph and oriented graph) and solve simple problems;
9) formulate the necessary and sufficient condition for a graph to be a Euler graph and solve simple problems by means of this condition; and
10) describe an historical problem related to the topics of the course or introduce persons connected with addition terms and proving of results.

Learning Content

Principle of mathematical induction: the principle, basis and step of induction. Solving problems by means of the principle of mathematical induction.

Elements of combinaorics: addition and multiplication rule. permutations without repetitions, variations and combinations and their properties. Use of addition and multiplication rules to solve problems. Permutations with repetitions, variations and combinations.

Introduction to graph theory: a vertex and an edge of a graph. Counting edges, degree of a vertex. Theorem: any graph has an even number of odd degree vertexes. Euler graph. Coherent graph. Coherence components of a non-coherent graph. The necessary and sufficient condition for a graph to be a Euler graph. Tree graph. Equivalent conditions for a graph to be a tree graph. Oriented graph.

3.7. Optional Course: 'Planimetry Part 1: Geometry of Polygons and Circles.'

Learning and Educational Objectives

The objective of the optional course is that students:

1) know basic terms and main results of triangles and circles geometry and are able to use the main methods to prove them (parallelism, congruence, similarity and a method of angles at the circumference);
2) are able to use the methods learnt to solve typical problems involving classical synthetic geometry and produce correct drawings; and
3) develop creative and flexible mathematical thinking.

Description of Course

The course consists of three basic fields:

1) parallel lines;
2) congruence and similarity of triangles; and
3) angles and line segments related to circumference and intersection and tangency of circumferences.

Learning Outcomes

At the end of the course, student can:

1) define the term 'parallelism of lines', formulate properties of parallelism and prove them;
2) use properties of parallelism and the ray's theorem to solve typical and (proving) problems;
3) define the terms 'congruence of triangles' and 'similarity of triangles', formulate the properties of congruence and similarity and prove the properties;
4) use the methods of congruence and similarity to solve (proving) problems;
5) formulate and prove the theorem of a height drawn from the vertex of the right angle of a right-angled triangle and its conclusions (Pythagorean Theorem, Euclid's Theorem and the theorem of height) and the converse theorem of Pythagoras' Theorem;
6) explain the difference between the terms 'congruence of triangles' and 'equiareality of triangles' and solve problems with relevant content;
7) know the inequalities of a triangle and use them to solve (proving) problems;
8) know the main results involving the size of an angle at the circumference and the angle between a chord and tangent of a circumference and use them to solve (proving)problems;
9) formulate and prove the theorems of two chords, a secant, a tangent and a secant and a tangent of a circumference and use the results to solve (proving)problems; and

10) solve simple (proving) problems involving the intersection and tangency of circumferences.

Learning Content

Parallel lines. Parallelism of lines. Parallelism properties of lines. Ray's theorem. Historical overview of problems related to parallelism of lines (the problem of the 'postulate of parallels').

Triangle. Definitions and properties of congruence and similarity of triangles. Theorem of a height drawn from the vertex of the right angle of a right-angled triangle and its conclusions (Pythagorean Theorem, Euclid's Theorem and the theorem of height). Converse theorem of Pythagoras' Theorem. Equiareality of triangles. Inequality of a triangle.

Circle and circumference. Central angel and angle at the circumference. Size of an angle at the circumference. Thales' Theorem. Angle between a chord and a secant. Theorems of two chords, two secant and a secant and a tangent of a circumference. Equality of tangent segments drawn from a given point to a circumference. Potency of a point a circumference. Interior (exterior) tangency of two circumferences.

3.8. Optional Course: 'Planimetry Part 2: Geometry of Polygons and Circles.'

Learning and Educational Objectives

The objective of the optional course is that students:

1) know the basic terms of geometry, main results of polygons and circles and are able to use main methods to prove them (parallelism, congruence, similarity, a method of angles at the circumference and a method of additional constructions);
2) are able to creatively use the methods learnt to solve (proving) problems of synthetic geometry and perform simple drawings correctly by means of compass and a ruler and/or a ICT tools using a programme of dynamic geometry; and
3) develop creative and flexible mathematical thinking.

Description of Course

The course consists of four basic fields:

1) classification and main properties of polygons (quadrangles);
2) cyclic quadrilateral;
3) line segments (midlines, medians, bisectors, heights and mediators) and circumferences (inscribed and circumscribed circle) related to a triangle; and
4) construction problems.

Learning Outcomes

At the end of the course, student can:

1) deduce formulas to find the sum of the interior and exterior angles of a polygon and the number of diagonals and use them to solve (proving) problems ;
2) define the terms 'congruence of polygons' and 'similarity of polygons' and use the methods of congruence and similarity to solve (proving) problems;
3) know the definitions and properties of quadrangles (square, rectangle, rhombus, parallelogram and trapezoid) and use them to solve (proving)problems;

4) formulate and prove the necessary and sufficient conditions for a quadrangle to be a cyclic quadrilateral and use the method of cyclic quadrilateral to solve (proving) problems and to justify the position of the four points on a circumference;
5) define the terms of the segments (midline, median, bisector, height and mediator) related to a triangle and prove their main properties and use the results to solve (proving)tasks;
6) use different methods to prove that the three medians (bisector, mediator and height) of any triangle intersect at a point;
7) know cross point of given segments where locate centers of the interior and exterior circumference of a triangle and use this to solve (proving)problems; and
8) achieve a certain skills for solving basic construction problems by means of compass and a ruler.

Learning Content

Polygons: convex and non-convex polygons and regular polygons. Sum of interior and exterior angles of a polygon. Diagonals of polygons. Congruence and similarity of polygons. Necessary and sufficient conditions for a quadrangle to be a square (rectangle, rhombus, parallelogram or trapezoid).
Cyclic quadrilateral. Necessary and sufficient conditions for a quadrangle to be an inscribed quadrangle: angles at the circumference resting on the same arc, angles at the circumference positioned opposite to each other, product of the lengths of the segments of diagonals (the property of intersecting chords of a circumference) and Ptolemy's Theorem. Four points positioned on a circumference.
Segments and circumferences in a triangle: midline of a triangle and the properties of the midlines and triangles formed from these segments. Necessary and sufficient condition for a point to be located on a bisector of a given angle (on the mediator of a given segment). Theorems on intersection of the medians of a triangle (bisectors, heights and mediators) at a given point. Interior and exterior circumference of a triangle.
Construction problems. Basic construction by means of compass and a ruler (bisector of a given angle, mediator of a segment, construction of a perpendicular or a parallel line on a line from a given point, construction of an interior and exterior circumference of a triangle, construction of a tangent of a circumference, division of a segment in a given ratio and construction of polygons). Historical overview of the possibility of performing classical construction problems (quadrature of a circle, duplication of a cube and trisection of an angle).